ELECTRICITY AND MAGNETISM,
RE-EXAM PART I AND II,
10 ECTS, 12/07/2017

#QUESTIONS: 4, #POINTS: 120

> Write your name and student number on every sheet. Use a separate sheet for each problem. Write clearly. Use of a (graphing) calculator is allowed. For all problems you have to write down your arguments and the intermediate steps in your calculations. You may make use of the formula sheet. The same notation is used as in the book, i.e. a bold-face $\mathbf{A}$ is a vector, T is a scalar. When you are asked to find a vector, do not forget about its direction! The bonus points will be added to the total points scored so that you can compensate a deficit in knowing one question with excellent knowing another one.

Question 1 - 30 points

1A. Consider the following situation. We place four identical negatively charged particles at the corners of a square with all sides a. What is the kinetic energy of the each of the charges, if we let them all fly away simultaneously? (10 points)

1B. Find the energy of a uniformly charged spherical shell of total charge q and radius R. (10 points)

1C. A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration. (10 points)

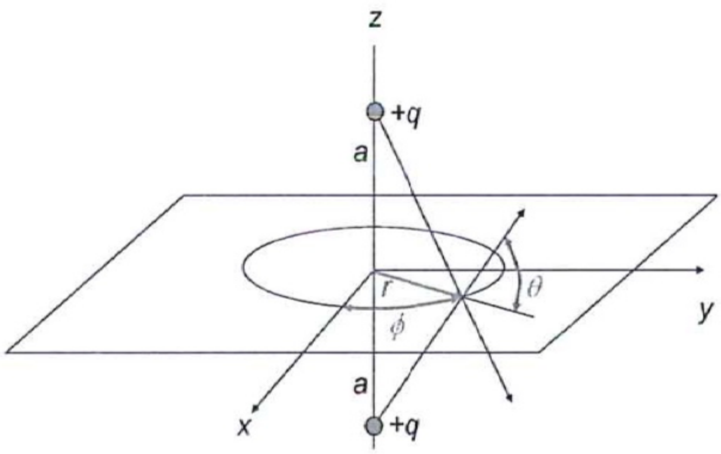
Question 2 - 30 points

2A. A long straight wire, carrying a uniform line charge λ, is surrounded by rubber insulation out to a radius a. Find the electric displacement everywhere, and the electric field outside the insulation. (10 points)

2B. A short circular cylinder of radius a and length L carries a "frozen-in" uniform magnetization $\mathbf{M}$ parallel to its axis. Find the bound current, and sketch the magnetic field of the cylinder. Make three sketches: one for $L \gg a$, one for $L \ll a$, and one for $L \sim a$. (10 points)

2C. For the bar magnet of question 2B, make careful sketches of $\mathbf{M}$, $\mathbf{B}$ and $\mathbf{H}$, assuming L is about $2a$. (10 points)

Question 3 - 30 points



Consider two equal point charges q, separated by a distance $2a$ as shown in the figure. The equidistant plane (i.e. where the distances between this plain and each charge in the set are equal) is the xy plane.

3A. Calculate the electric field, the magnetic field, and the Poynting vector in terms of q, r, and ϕ (but not θ - express it via other variables) in the equidistant plane. (11 points)

3B. Find the T_{zz}, T_{xz} and T_{yz} components of the Maxwell stress tensor in the equidistant plane. (9 points)

3C. Determine the force on the upper charge using the Maxwell stress tensor. (8 bonus points)

3D. Does your result make sense? (2 bonus points)
(Tip: you might find useful the following integral: $\int_0^\infty \frac{r^3}{(r^2+a^2)^3} dr = \frac{1}{4a^2}$)

Question 4 - 30 points

Albert Einstein wondered at age 16 what the fields **E** and **B** are if you would travel together with the plain electromagnetic wave. He deduced it at age 26 when he applied the Lorentz transformation to the electromagnetic field. Following Einstein, find the electromagnetic field if you travel along the electromagnetic wave.

4A. Write down the electric and magnetic fields of a sinusoidal electromagnetic plane wave of angular frequency ω and amplitude E_0, and polarized into the y-direction, which is travelling into the x-direction through the vacuum. (4 points)

4B. The same wave is observed from an inertial frame $\bar{\mathcal{S}}$, moving with the speed v in the x-direction. Find all (i.e. x, y, z) components the electric and magnetic fields in $\bar{\mathcal{S}}$ using the following substitution: $\alpha = \gamma(1 - \frac{v}{c})$. (8 points)

4C. What is the ratio of the intensity $\bar{I}$ in $\bar{\mathcal{S}}$ to the intensity I in $\mathcal{S}$? (5 points)

4D. Now v approaches c. What are the field amplitudes and intensity of the electromagnetic wave? (Tip: be careful as when $v \to c, \gamma \to \omega$(3 points)

4E. Express the electric and magnetic fields in $\bar{\mathcal{S}}$ in terms of $\bar{\mathcal{S}}$ coordinates as $\bar{\mathbf{E}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ and $\bar{\mathbf{B}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$.What is the frequency ω of the wave in $\bar{\mathcal{S}}$ in terms of v and c?

You might need the Lorentz transformations of the coordinate and time: $x = \gamma(\bar{x} + v\bar{t})$; $t = \gamma(\bar{t} + \frac{v}{c^2}\bar{x})$; $z = \bar{z}$. Express the parameters of the wave in $\bar{\mathcal{S}}$ (amplitude, frequency, and wavevector) in terms of α. (10 bonus points)